

# Participant number fluctuations for higher moments of a multiplicity distribution

Viktor Begun<sup>1,\*</sup>

<sup>1</sup>*Institute of Physics, Jan Kochanowski University, PL-25406 Kielce, Poland*

The independent participant model is generalized for skewness and kurtosis. The obtained relations allow to calculate the fluctuations of an arbitrarily high order. From the comparison with the SPS and the LHC data it is found that the participants are not nucleons. The contribution of the participant fluctuations increases with the order of fluctuations. The 5% centrality bins selected for the analysis at the LHC by ALICE seems to be too large. The fluctuations measures are dominated by the fluctuations of participants there. The method to quantify the value of participant number fluctuations experimentally is proposed.

PACS numbers:

Keywords: wounded nucleons, independent participants, independent sources, participant fluctuations, scaled variance, skewness, kurtosis, higher moments

Many observables in high energy collisions scale with the number of participants -  $N_P$  - the nucleons that interacted inelastically during a collision. The total number of charged particles is proportional to  $N_P$  in the measured energy range [1–3]. This effect was addressed first in Ref. [4] within the wounded nucleon model. It is based on the optical limit of the famous Glauber model [5, 6], which physically means that the wounded nucleons emit particles independently from each other. The latter is more general requirement the same as in the independent source model or independent participant model. The term participants is the most commonly used now, therefore it is used later on in this paper.

It was found that the behavior of the scaled variance of a multiplicity distribution as the function of  $N_P$  can be also qualitatively explained by the fluctuations of participants [7, 8]. The scaled variance is proportional to the second moment of a multiplicity distribution. The ratio of the fourth to the square of the second moment is called kurtosis. The STAR collaboration observes the non-monotonous behavior of the normalized kurtosis for the net-proton distribution [9, 10]. This might be an indication of the critical point of strongly interacting matter, as higher moments of fluctuations are more sensitive to the proximity of the QCD critical point [11]. It is quite intriguing that the kurtosis has a minimum in the vicinity of the collision energy where the NA49 and the STAR collaborations see the famous  $K^+/\pi^+$  horn [12–15]. A significant effort of the NA61/SHINE collaboration, the successor of the NA49, is going to be devoted to the study of high order fluctuations. A most challenging background for these studies seems to be the fluctuations of nucleon participants, similar to the case with the scaled variance. These fluctuations are experimentally unavoidable and, therefore, should be reliably estimated. One can derive the necessary formulas for the third moment from the Ref. [16], see also [17]. However, it seems that, in spite of the practical importance, the influence of the fluctuations of nucleon participants for higher moments was not considered yet.

In the present paper the expressions for the third and the fourth moments are written explicitly for arbitrary distributions of both measured particles and the participants. The only assumptions are that the participants are *identical* and *independent*. The proposed method of calculation allows to derive straightforwardly the influence of the participant fluctuations for arbitrarily high moments.

The multiplicity of some particles  $N$  created in a collision is the sum of the contributions from  $N_P$  participants

$$N = n_1 + n_2 + \dots + n_{N_P}. \quad (1)$$

The number of particles  $n_i$  from a participant  $i$  fluctuates. If the participants are identical, then the average  $\langle n_i \rangle = \langle n_j \rangle = \langle n_1 \rangle$  and

$$\langle N \rangle = \sum_{N_P} P(N_P) \left\langle \sum_{i=1}^{N_P} n_i \right\rangle = \sum_{N_P} P(N_P) N_P \langle n_1 \rangle = \langle N_P \rangle \langle n_1 \rangle, \quad (2)$$

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\*Electronic address: [viktor.begun@gmail.com](mailto:viktor.begun@gmail.com)

where  $P(N_P)$  is the probability distribution of the participants number. Similarly

$$\begin{aligned}\langle N^2 \rangle &= \sum_{N_P} P(N_P) \left\langle \left( \sum_{i=1}^{N_P} n_i \right)^2 \right\rangle \\ &= \sum_{N_P} P(N_P) \left( \sum_{i=1}^{N_P} \langle n_i^2 \rangle + \sum_{i \neq j=1}^{N_P} \langle n_i n_j \rangle \right) \\ &= \langle N_P \rangle \langle n_1^2 \rangle + \langle N_P(N_P - 1) \rangle \langle n_1 \rangle^2 ,\end{aligned}\quad (3)$$

where the assumption that the participants are independent  $\langle n_i n_j \rangle = \langle n_i \rangle \langle n_j \rangle = \langle n_1 \rangle^2$  is used. Equations (2) and (3) give the famous formula for the scaled variance,

$$\omega = \frac{\langle N^2 \rangle - \langle N \rangle^2}{\langle N \rangle} = \omega_1 + \langle n_1 \rangle \omega_P , \quad (4)$$

which is present already in [4]. It is the sum of the fluctuations from one participant  $\omega_1$  and the fluctuations of participant number  $\omega_P$  times the mean multiplicity of particles of interest from one participant  $\langle n_1 \rangle$ . Using the multinomial theorem,

$$\left( n_1 + n_2 + \dots + n_{N_P} \right)^k = \sum_{k_1, k_2, \dots, k_{N_P}} \frac{k!}{k_1! k_2! \dots k_{N_P}!} n_1^{k_1} n_2^{k_2} \dots n_{N_P}^{k_{N_P}} \delta \left( k - \sum_{i=1}^{N_P} k_i \right) , \quad (5)$$

where  $\delta$  is the Kronecker delta function, one can obtain arbitrarily high moment in the model of independent participants. For the third and the fourth moments one has:

$$\begin{aligned}\langle N^3 \rangle &= \langle N_P \rangle \langle n_1^3 \rangle \\ &\quad + 3 \langle N_P(N_P - 1) \rangle \langle n_1^2 \rangle \langle n_1 \rangle \\ &\quad + \langle N_P(N_P - 1)(N_P - 2) \rangle \langle n_1 \rangle^3 ,\end{aligned}\quad (6)$$

$$\begin{aligned}\langle N^4 \rangle &= \langle N_P \rangle \langle n_1^4 \rangle \\ &\quad + 4 \langle N_P(N_P - 1) \rangle \langle n_1^3 \rangle \langle n_1 \rangle \\ &\quad + 3 \langle N_P(N_P - 1) \rangle \langle n_1^2 \rangle^2 \\ &\quad + 6 \langle N_P(N_P - 1)(N_P - 2) \rangle \langle n_1^2 \rangle \langle n_1 \rangle^2 \\ &\quad + \langle N_P(N_P - 1)(N_P - 2)(N_P - 3) \rangle \langle n_1 \rangle^4 .\end{aligned}\quad (7)$$

The coefficients in front of the  $\langle n_1^{k_i} \rangle^{k_j}$  terms are given by the product of the multinomial coefficient, the number of permutations  $\frac{N_P!}{(N_P - k_i)!}$ , and the additional degeneracy factor that appears due to the fact that the emitted particles are indistinguishable. For example, the factor before  $\langle n_1 \rangle^4$  in (7) is equal to the multinomial coefficient  $\frac{4!}{1!1!1!1!} = 4!$  times the number of ways to pick up four different participants  $\frac{N_P!}{(N_P - 4)!}$ , divided by the degeneracy factor  $4!$  due to the replacement  $\langle n_i \rangle \langle n_j \rangle \langle n_k \rangle \langle n_l \rangle = \langle n_1 \rangle^4$ . The coefficient in front of  $\langle n_1^2 \rangle \langle n_1 \rangle^2$  in (7) is equal to  $\frac{4!}{2!1!1!0!} = 12$  times  $\frac{N_P!}{(N_P - 3)!}$ , divided by  $2!$  due to  $\langle n_i \rangle \langle n_j \rangle = \langle n_1 \rangle^2$ , etc.. The sum of all the coefficients for  $\langle n_1^{k_i} \rangle^{k_j} = 1$  before the averaging over participants gives  $N_P^k$ , which can be used for a quick check. The formulas for higher moments can be derived in the similar way. The raw moments  $\langle N^k \rangle$  are directly related to central moments of a distribution  $P(N)$

$$m_k = \sum (N - \langle N \rangle)^k P(N) . \quad (8)$$

The second, the third, and the fourth moments in the model of independent participants equal to:

$$m_2 = \langle N_P \rangle m_2^1 + \langle n_1 \rangle^2 m_2^P , \quad (9)$$

$$m_3 = \langle N_P \rangle m_3^1 + \langle n_1 \rangle^3 m_3^P + 3 \langle n_1 \rangle m_2^P m_2^1 , \quad (10)$$

$$\begin{aligned}m_4 &= \langle N_P \rangle (m_4^1 - 3(m_2^1)^2) + 3m_2^P (m_2^1)^2 + 4 \langle n_1 \rangle m_2^P m_3^1 + 6 \langle n_1 \rangle^2 m_3^P m_2^1 \\ &\quad + \langle n_1 \rangle^4 [m_4^P - 3(m_2^P)^2] + 3(m_2^P)^2 ,\end{aligned}\quad (11)$$

where  $m_k^1$  and  $m_k^P$  are defined the same as  $m_k$  (8) for the distribution of particles produced by one source  $P(n_1)$  and for the distribution of participants  $P(N_P)$ .

The combination of central moments gives the scaled variance, the normalized skewness, and the normalized kurtosis:

$$\omega = \frac{m_2}{\langle N \rangle}, \quad S\sigma = \frac{m_3}{m_2}, \quad \kappa\sigma^2 = \frac{m_4}{m_2} - 3m_2, \quad \text{where } \sigma^2 = m_2. \quad (12)$$

They describe the width, the asymmetry, and the sharpness of a distribution with a single maximum, correspondingly. Skewness and kurtosis are much more sensitive to the properties of a multiplicity distribution. A Poisson distribution has  $\omega = S\sigma = \kappa\sigma^2 = 1$  for the same mean multiplicity, while  $\omega$  is a free parameter and  $S\sigma = \kappa\sigma^2 = 0$  for Normal (Gauss) distribution. In the independent participant model the normalized skewness equals

$$S\sigma = \frac{\omega_1 S_1\sigma_1 + \langle n_1 \rangle \omega_P [3\omega_1 + \langle n_1 \rangle S_P\sigma_P]}{\omega_1 + \langle n_1 \rangle \omega_P}, \quad (13)$$

and normalized kurtosis:

$$\kappa\sigma^2 = \frac{\omega_1 \kappa_1\sigma_1^2 + \omega_P [\langle n_1 \rangle^3 \kappa_P\sigma_P^2 + \langle n_1 \rangle \omega_1 (3\omega_1 + 4S_1\sigma_1 + 6\langle n_1 \rangle S_P\sigma_P)]}{\omega_1 + \langle n_1 \rangle \omega_P}. \quad (14)$$

Scaled variance, skewness and kurtosis depend crucially on the strength of participant fluctuation  $\omega_P$ . If it is zero, then the information about participants is left in the mean multiplicity, but is cancelled in fluctuations:

$$\langle N \rangle = \langle N_P \rangle \langle n_1 \rangle, \quad \omega = \omega_1, \quad S\sigma = S_1\sigma_1, \quad \kappa\sigma^2 = \kappa_1\sigma_1^2, \quad \text{for } \omega_P = 0, \quad (15)$$

so that one observes the fluctuations from one source. It is a desired situation, because participant fluctuations are mainly driven by the uncertainty of the centrality determination. They may mimic or hide the QCD critical point and any other signal. The fluctuations of participants seem to be unavoidable, because one always has a finite centrality window in experiment. If this window is too narrow, then one may cut also the fluctuations from one source. Therefore, one should find the balance between fluctuations of participants  $\omega_P$ , the number and fluctuations of particles from one participant  $\omega_1/\langle n_1 \rangle$ .

For small fluctuations of the participants,  $\omega_P, S_P\sigma_P \ll \omega_1/\langle n_1 \rangle$  and  $\kappa_P\sigma_P^2 \ll \omega_1^2/\langle n_1 \rangle^2$ , one obtains:

$$\omega \simeq \omega_1, \quad S\sigma \simeq S_1\sigma_1 + 3\langle n_1 \rangle \omega_P, \quad \kappa\sigma^2 \simeq \kappa_1\sigma_1^2 + \langle n_1 \rangle \omega_P (3\omega_1 + 4S_1\sigma_1), \quad (16)$$

i.e., the scaled variance is determined by the fluctuations from one participant, however skewness and kurtosis further depend on how large is the product  $\langle n_1 \rangle \omega_P$  compared to the skewness and kurtosis for one source. The fluctuations from one source should be large close to critical point or phase transition. For example, all moments higher than  $k > 2$  diverge at Bose-Einstein condensation [18], which is the third order phase transition.

For large enough fluctuations of the participants,  $\omega_P \gg \omega_1/\langle n_1 \rangle$ ,  $\omega_P \gg \kappa_1\sigma_1^2/(\langle n_1 \rangle \omega_1)$ , and  $S_P\sigma_P \gg \omega_1/\langle n_1 \rangle$ ,  $S_P\sigma_P \gg S_1\sigma_1/\langle n_1 \rangle$  one finds:

$$\omega \simeq \langle n_1 \rangle \omega_P, \quad S\sigma \simeq \langle n_1 \rangle S_P\sigma_P + 3\omega_1, \quad \kappa\sigma^2 \simeq \langle n_1 \rangle^2 \kappa_P\sigma_P^2 + 6\langle n_1 \rangle \omega_1 S_P\sigma_P, \quad (17)$$

i.e., the observed fluctuations are determined mainly by the fluctuations of the participants. Note the  $\langle n_1 \rangle$  and  $\langle n_1 \rangle^2$  multipliers in front of scaled variance, skewness and kurtosis from participants in (17). For large energies  $\langle n_1 \rangle$  grows fast and leads to the domination of participant fluctuations for high moments even for relatively small  $\omega_P$ ,  $S_P\sigma_P$  and  $\kappa_P\sigma_P^2$ . The participant fluctuations are rather large in a standard centrality interval. A finer centrality selection [19] or (and) special variables should be used to cancel the fluctuations of participants [20–22].

The experimental information on participant fluctuations is quite ambiguous. The behavior of the scaled variance of a multiplicity distribution in nucleus-nucleus (A+A) collisions as the function of  $N_P$  was qualitatively explained by the fluctuations of participants both at SPS and at RHIC [7, 8]. However, more recent data of NA49 and NA61/SHINE [23–25] show that

$$\omega_{Pb+Pb}^{\text{ch}} < \omega_{p+Pb}^{\text{ch}} < \omega_{p+p}^{\text{ch}} \quad \text{at SPS}, \quad (18)$$

while one would expect the opposite dependence from the participant model. Using Eq. (4) one obtains

$$\omega_{Pb+Pb}^{\text{ch}} = \omega_1 + \langle n_{Pb+Pb}^{\text{ch}} \rangle \omega_P, \quad \text{where } \langle n_{Pb+Pb}^{\text{ch}} \rangle = \frac{\langle N_{Pb+Pb}^{\text{ch}} \rangle}{\langle N_P \rangle}, \quad (19)$$

and  $\langle n_{\text{Pb+Pb}}^{\text{ch}} \rangle$  is the number of charged particles per participant. One can see from Eqs. (18) and (19) that the fluctuations in Pb+Pb and p+Pb can not be constructed from fluctuations of p+p. Both  $\langle n_{\text{Pb+Pb}}^{\text{ch}} \rangle$  and  $\omega_{\text{P}}$  are positive, therefore, if  $\omega_1 = \omega_{\text{p+p}}^{\text{ch}}$  in (19), then fluctuations of participants,  $\omega_{\text{P}}$ , must be negative in this case. It is impossible, since  $\omega$  is positive by definition. However, the fluctuations in p+p and in Pb+Pb are similar at SPS, therefore the relation (18) may be attributed to a combination of some other effects.

The situation should be clear at the LHC, because p+p roughly follow the KNO scaling, which leads to  $\omega_{\text{p+p}}^{\text{ch}} \sim \langle N_{\text{p+p}}^{\text{ch}} \rangle$  and a fast rise of fluctuations with increasing the energy of the collision,  $\sqrt{s_{\text{NN}}}$ , while for A+A a weaker dependence of fluctuations with energy is expected [26]. The ALICE collaboration has published the results for fluctuations of charged particles  $\omega^{\text{ch}}$  in Pb+Pb collisions within the  $|\eta| < 0.8$  rapidity range. Their comparison with the AMPT and HIJING string transport models shows different fluctuations and the different dependence on  $\langle N_{\text{P}} \rangle$  than in the experiment [27].

Instead of running a transport code one may solve the inverse task. Namely, determine how large should be the fluctuations of the participants in order to describe the data, assuming different fluctuations of the participants. The CMS and the ALICE collaborations have published the data for fluctuations in p+p [28, 29], as well as the rapidity distributions of charged particles, and the number of participants at different centralities in Pb+Pb at  $\sqrt{s_{\text{NN}}} = 2.76$  TeV [3, 30]. Therefore, one can check whether the fluctuations in Pb+Pb is the sum of the fluctuations in p+p and the fluctuations of participants. One should take the measured fluctuations in Pb+Pb,  $\omega_{\text{Pb+Pb}}^{\text{ch Acc}}$ , from ALICE [27]. Calculate the rate of how many charged particles are accepted within their rapidity window,  $|\eta| < 0.8$ , with respect to the number of charged particles in the full rapidity  $q = \langle N_{\text{Pb+Pb}}^{\text{ch}} \rangle|_{|\eta| < 0.8} / \langle N_{\text{Pb+Pb}}^{\text{ch}} \rangle$ . Then one should use the well known acceptance formula for scaled variance, see e.g. Ref. [31],

$$\omega^{\text{Acc}} = 1 - q + q\omega, \quad (20)$$

to reconstruct the fluctuations in the full rapidity range,  $\omega = \omega_{\text{Pb+Pb}}^{\text{ch}}$ , then use Eq. (19), and find

$$\omega_{\text{Pb+Pb}}^{\text{ch Acc}} = 1 - q + q\omega_{\text{Pb+Pb}}^{\text{ch}} = \omega_1^{\text{Acc}} + \langle n_{\text{Pb+Pb}}^{\text{ch Acc}} \rangle \omega_{\text{P}}, \quad (21)$$

where  $\omega_1^{\text{Acc}} = 1 - q + q\omega_1$  and  $\langle n_{\text{Pb+Pb}}^{\text{ch Acc}} \rangle = q\langle n_{\text{Pb+Pb}}^{\text{ch}} \rangle$ . The fluctuations in p+p equal to  $\omega_{\text{p+p}}^{\text{Acc}} \simeq 4.6, 8.46, 11.36, 13.74$  in the rapidity intervals  $|\Delta\eta| = 0.5, 1., 1.5, 2.4$ , correspondingly, therefore,

$$\omega_1^{\text{Acc}} = \omega_{\text{p+p}}^{\text{Acc}}|_{|\Delta\eta| < 0.8} \simeq 7. > \omega_{\text{Pb+Pb}}^{\text{ch Acc}} \simeq 3., \quad (22)$$

and the fluctuations of the participants are negative in (21), similar to that at the SPS (18). The acceptance  $q \simeq 15\%$  in Pb+Pb at ALICE. It gives  $\omega_{\text{p+p}} = (\omega_{\text{p+p}}^{\text{Acc}}|_{|\Delta\eta| < 0.8} - 1 + q)/q \simeq 41$  for the whole acceptance. One may argue that some processes may damp the fluctuations from one participant in Pb+Pb compared to p+p. Let us pick up some numbers in order to quantify a possible outcome and consider three cases.

First, the fluctuations from one source equal to the maximal measured fluctuations in p+p,  $\omega_1 = \omega_{\text{p+p}}^{\text{ch}}|_{|\eta| < 2.4} = 13.74$ . Let's call this case 'Maximal'.

Second, the fluctuations from one source are Poisson-like,  $\omega_1 = \omega_{\text{Poisson}} = 1$ , called 'Poisson'. For these two cases we know all the terms in Eq. (21), except for  $\omega_{\text{P}}$ , which is calculated from (21).

Third case – we do not know the fluctuations from one source, but we know that the fluctuations of participants are of the order of unity,  $\omega_{\text{P}} = 1$ , as in HIJING and AMPT in Ref. [27], and then calculate  $\omega_1$  from (21). This case is called 'Transport'.

The results are shown in Figs. 1 and 2. The continuous lines in Fig. 1 and in Fig. 2 left are the fit of the ALICE data for  $\omega_{\text{Pb+Pb}}^{\text{ch Acc}}$ . They go through the data points by definition. The dashed and dash-dotted lines is the decomposition of  $\omega_{\text{Pb+Pb}}^{\text{ch Acc}}$  into two parts according to Eq. (21), right. The corresponding fluctuations of the participants are shown in Fig. 2 right.

The acceptance slowly grows with centrality in the 2.76 TeV Pb+Pb collisions at the LHC. Therefore, the 'Maximal' fluctuations from one source,  $\omega_1^{\text{Acc}}$ , also grow, while the fluctuations of participants must decrease fast, because the total fluctuations decrease, see Eq. (21). The 'Maximal' fluctuations from one source are above the experimental measurements for large  $\langle N_{\text{P}} \rangle$ , therefore, the fluctuations of participants,  $\omega_{\text{P}}$ , become negative, which is forbidden by the definition of  $\omega$ . The absolute value of participant fluctuations is small, so that the measures are done in the 'good' limit (16). However,  $\omega_{\text{P}}$  is too small for the LHC, and even smaller than at RHIC, compare the dash-dotted line in Fig. 2 with Fig. 1 from [8].

For 'Poisson' fluctuations from one source the acceptance dependence is cancelled,  $\omega_1^{\text{Acc}} = \omega_1 = 1$  according to Eq. (20), and the fluctuations of participants are similar to that at RHIC. However, one expects a strong growth of the participant fluctuations with energy from transport models [21]. Moreover, the measurements at

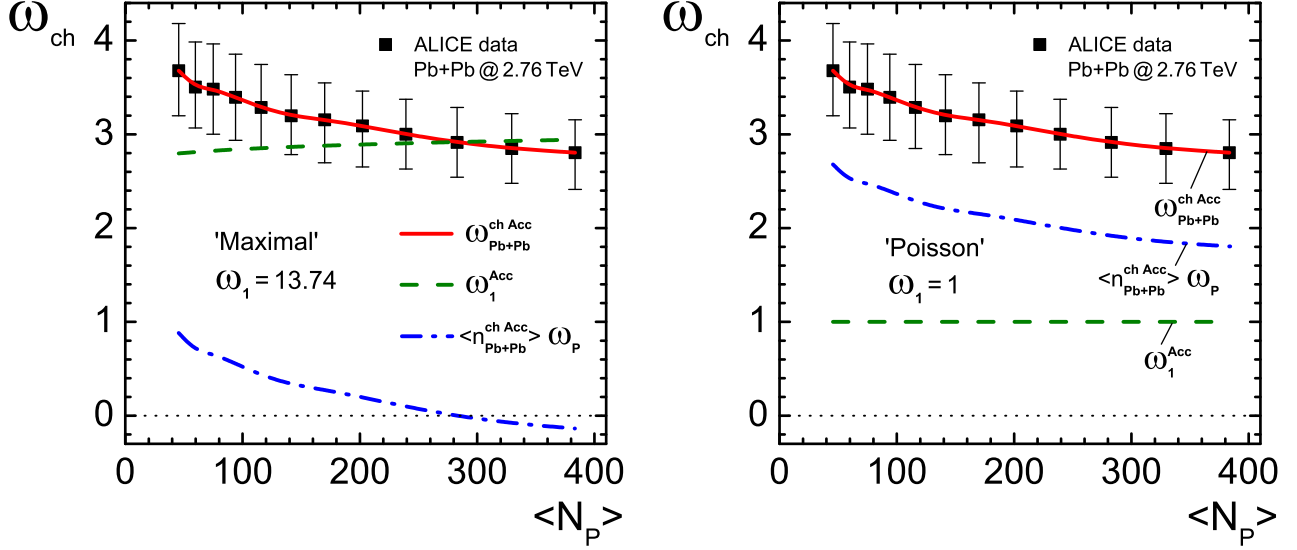


FIG. 1: The decomposition of experimentally measured fluctuations of charged particles  $\omega_{Pb+Pb}^{ch Acc}$  as the function of the number of participants  $\langle N_P \rangle$  [27] on the fluctuations due to the fluctuations of participants  $\langle n_{Pb+Pb}^{ch Acc} \rangle \omega_P$  and due to the fluctuations from one participant  $\omega_1^{Acc}$  in the measured acceptance, assuming some value of the fluctuations from one participant  $\omega_1$  in the full acceptance.

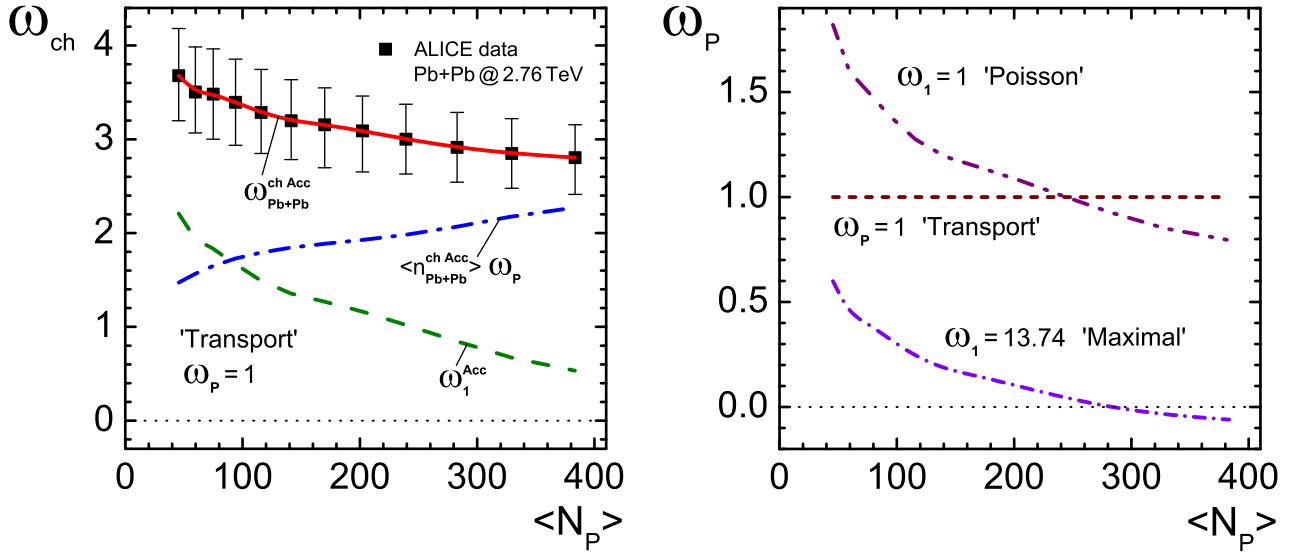


FIG. 2: Left: The same as Fig. 1 for the fluctuations of participants  $\omega_P = 1$ . Right: The extracted fluctuations of participants, assuming some value of the fluctuations from one source  $\omega_1$  in the full acceptance.

ALICE are done in the 'bad' limit, when all the measures are determined by the fluctuations of the participants, see Eq. (17).

The 'Transport' case is in between the 'Maximal' and the 'Poisson', closer to the 'Poisson'. The measurement are done in the 'bad' limit (17), when the fluctuations of participants determine the results.

One may conclude that the independent participant model can describe fluctuations of charged particles in Pb+Pb at the LHC only if the fluctuations from one participant,  $\omega_1$ , are much smaller than the fluctuations of charged particles in p+p reactions. Therefore, the participants are not nucleons.

If the fluctuations of participants are larger than Poisson,  $\omega_P \geq 1$ , moreover, if they are as large as predicted by transport models, then the 5% centrality bins selected for the analysis at the LHC by ALICE are too large. In this case the fluctuations measures are dominated by the fluctuations of participants and by the corresponding

experimental limitations, like the uncertainty in the centrality determination.

There are many ways to look for a possible solution. The participants can be quarks, then the number of particles from one source,  $\langle n_{\text{Pb+Pb}}^{\text{ch Acc}} \rangle$ , reduces three times, since there are three quarks in each nucleon. It leads to the increase of the  $\omega_{\text{P}}$  three times, in order to keep the same value of the product  $\langle n_{\text{Pb+Pb}}^{\text{ch Acc}} \rangle \omega_{\text{P}}$  in (21). Another possibility is that the sources are not identical and/or strongly correlated. The examination of these possibilities requires further theoretical studies and more data.

One should check experimentally whether participant model works for fluctuation, eliminate the fluctuations of participants, and obtain the fluctuations from one source. In order to do that, one should consider the most central collisions, reduce the centrality window, and check how the fluctuations change, taking, let say,  $c = 0-20\%$ ,  $c = 0-15\%$ ,  $c = 0-10\%$ ,  $c = 0-5\%$ ,  $c = 0-2.5\%$ ,  $c = 0-1\%$ , etc.. If the participant model works, then one would expect a fast decrease of the fluctuations due to the decrease of the participant fluctuations. The decrease should slow down at some centrality, which is narrow enough, so that the participant fluctuations do not contribute. If the remaining fluctuations are not already Poisson-like due to very small acceptance, then these are the fluctuations from one source.

It seems that the amount of participant fluctuations should be determined before measurements of the higher moments, because participants fluctuations may be strong enough to mimic or hide any other effect.

### Acknowledgments

The author thanks to N. Xu for encouraging discussions at WPCF 2015 and CPOD 2016 conferences, and to M. I. Gorenstein, W. Broniowski, V. Koch, M. Praszalowicz, A. Rustamov, M. Rybczynski, and I. Selyuzhenkov, for fruitful comments and suggestions. This work was supported by Polish National Science Center grant No. DEC-2012/06/A/ST2/00390.

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